NUMERICAL ANALYSIS OF HEAT PUMP MODELS. COMPARATIVE STUDY BETWEEN EQUATION-FIT AND REFRIGERANT CYCLE BASED MODELS.

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Abstract

An equation-fit (EF) and a refrigerant cycle (RC) based heat pump models have been implemented, validated, analyzed and compared to each other under steady state conditions for a brine to water heat pump. Models validations have been provided through comparisons against experimental data obtained at ISFH. The advantages and disadvantages of the both models have been identified. This work provides significant inputs regarding the selection of a specific model depending on the needs. Analysis of mass flow rates and calculations far from typical catalogue data (non-standard conditions) are provided. The main conclusions can be summarized as: i) the EF model is recommended when the boundary conditions for the estimation and prediction modes are the same and when non-standard conditions are considered; ii) the RC model is the chosen alternative when the mass flow rates are modified from the estimation to the prediction mode.

1 Introduction

Heat pumps are becoming an important technology in the renewable energy field. Studies of capabilities and limitations of existing models in order to choose appropriately which models to use for specific situations is considered to be of importance. Validation and analysis of an equation-fit and a refrigerant cycle based models for a brine to water heat pump in heating mode is provided in this paper. A very important feature that the models must fullfill is that the necessary inputs are to be estimated only from catalogue data typically provided by manufacturers.

The so-called YUM model [1] has been selected as a representative of the equation-fit (EF) based models. A water/brine source heat pump parameter estimation model described in [2] is chosen to represent the refrigerant cycle (RC) based models.

The models have been implemented in two modes: i) estimation and ii) prediction. The estimation mode calculates the input parameters needed for the models using catalogue or experimental data. The prediction mode solves the heat pump model with defined inputs.

These models have been validated by the authors under the framework of IEA SHC Task 44 / HPP A38 : Solar and Heat Pumps in [3] using some commercial catalogue heat pumps data. The RC model was validated for scroll and reciprocating compressors using several refrigerants. Moreover a heat pump using a double circuit with two compressors were included in the analysis. In all analyzed cases, the estimation procedure of the EF model was proved to be easier and more accurate compared to the RC model, not matter which type of heat pump, refrigerant or compressor were used. Therefore, in order to calculate steady state conditions in normal catalogue data range, the EF model was shown to be the best alternative. In the present paper the validation and comparison between models is provided for different mass flow rates and under non-standard conditions using experimental data obtained at ISFH.

2 Mathematical formulation

2.1 Equation fit based model

The YUM model [1] is a black-box model based on quasi steady state performance maps. The mathematical formulation is simplified to a two-dimensional polynomial plane able to describe air and water/brine source heat pumps. This model is based on a biquadratic polynomial fit of the condenser heat power Q_c and the compressor work W_{cp}:

$$Q_c = bq_1 + bq_2\bar{T}_{e,in} + bq_3\bar{T}_{c,out} + bq_4\bar{T}_{e,in}\bar{T}_{c,out} + bq_5\bar{T}^2_{e,in} + bq_6\bar{T}^2_{c,out} \tag{1}$$

$$W_{cp} = bp_1 + bp_2\bar{T}_{e,in} + bp_3\bar{T}_{c,out} + bp_4\bar{T}_{e,in}\bar{T}_{c,out} + bp_5\bar{T}^2_{e,in} + bp_6\bar{T}^2_{c,out} \tag{2}$$

where $T_{e,in}$ is the fluid inlet temperature in the evaporator and $T_{c,out}$ the fluid outlet temperature in the condenser. The normalized temperature $\bar{T}$ is obtained from $\bar{T} = T[^oC]/273.15 + 1$. In the estimation mode, the polynomial coefficients are calculated using the multidimensional least square fitting algorithm of GSL (GNU Scientific library, [4]). In prediction model a brent solver [4] is employed.

2.2 Refrigerant cycle based model

The model solves the refrigerant circuit using simple models for evaporator, condenser, expansion valve and compressor. The inputs of the models are obtained by means of multidimensional parameter minimization from catalogue or experimental data. A reciprocating [2] and scroll [5] compressor models have been implemented to cover most of the heat pumps. Physical properties of refrigerants are calculated using a pre-processed matrix data obtained from NIST calculations to speed up the computational time. Moreover, a method to estimate the performance for different brine solutions has also been included in the present work as explained in [5].
The two heat exchangers are solved using the $\epsilon - NTU$ model [6] assuming negligible pressure lost. For a phase change process at constant temperature the efficiency of the heat exchanger ϵ can be obtained from:

$$\epsilon = 1 - e^{\frac{UA}{c_p \dot{m}}} \tag{3}$$

where the exponent term represents the number of transfer units NTU, UA is the global heat transfer coefficient in $[W/K]$, c_p is the fluid specific heat capacity in $[J/kgK]$ and $\dot{m}$ is the fluid mass flow rate in $[kg/s]$. Since this model uses only catalogue data, the configuration, length and other details of the heat exchangers are unknown. Therefore, the UA value is estimated from experiments or from catalogue data. Once the efficiency is obtained, the condensing and evaporating temperatures, T_c and T_e respectively, can be calculated:

$$T_e = T_{fe,i} - \frac{Q_e}{\epsilon c_p \dot{m}_e} \tag{4}$$

$$T_c = T_{fc,i} + \frac{Q_c}{\epsilon c_p \dot{m}_c} \tag{5}$$

where $T_{f,i}$ is the fluid inlet temperature in $[K]$, Q is the heat power in $[W]$ and the subscript e and c stand for evaporator and condenser respectively. At this stage, in prediction mode, Q_c and Q_e are

unknown, thereby an iterative procedure is needed. In the estimation mode these values are obtained from the experiments or catalogue data and no iterations are necessary.
In prediction mode, the heat in the evaporator is obtained from the refrigerant side as:

$$Q_e = \dot{m}_r(h_{re,out} - h_{re,in}) \tag{6}$$

Here, $\dot{m}_r$ is the refrigerant mass flow rate, $h_{re,out}$ and $h_{re,in}$ are the outlet and inlet enthalpy of the refrigerant in the evaporator in $[J/kg]$. The condenser heat is then obtained from the global heat balance of the heat pump:

$$Q_c = Q_e + W_{cp} \tag{7}$$

where W_{cp} is the compressor work. The enthalpy values used in Eq.6 are obtained from saturation values at the respective temperatures of the condenser and evaporator assuming and adiabatic expansion process. Moreover, the $h_{re,out}$ is actually neglecting the superheating effect but this should be compensated with and underpredicted UA_e value estimated by the model [2]. The same reasoning also applies to the neglected superheating and subcooling values of the condenser.
In order to calculate the compressor work needed in Eq.7 the following expression is used:

$$W_{cp} = \frac{W_{cp,t}}{\eta} + W_{loss} \tag{8}$$

where $W_{cp,t}$ is the theoretical compressor work, η the electro-mechanical efficiency and W_{loss} the constant part of the electro-mechanical power loss. The electro-mechanical parameters η and W_{loss} are inputs of the model and thereby calculated in the estimation mode.
The values of $\dot{m}_r$ of Eq.6 and $W_{cp,t}$ of Eq.8 are obtained from the compressor model, which is the key aspect in the RC based model. In this paper, only a heat pump with a scroll compressor has been analyzed.

2.2.1 Scroll compressor

The scroll compressor model has been described in [5]. The compressor mathematical description distinguishes between the external pressure ratio π defined as:

$$\pi = \frac{p_c}{p_e} \tag{9}$$

where p_c and p_e are the condensing and evaporating pressures in $[Pa]$, and the build-in pressure ratio π_i defined as:

$$\pi_i = \frac{p_{in}}{p_e} = \nu_i^{\gamma} \tag{10}$$

where the build-in volume ratio ν_i is an input of the model. In design conditions ($\pi = \pi_i$) the compressor work is calculated using the theoretical isentropic work [2]. For under-compression ($\pi < \pi_i$) and over-compression ($\pi > \pi_i$) the theoretical compressor work is higher than that of the isentropic process and can be calculated with:

$$W_{cp,t} = \frac{\gamma}{\gamma - 1} p_e \dot{m}_r \rho_{in} \left[\frac{\gamma - 1}{\gamma} \frac{\pi}{\nu_i} + \frac{\pi^{\frac{\gamma-1}{\gamma}}}{\gamma} - 1 \right] \tag{11}$$

where ρ_{in} is the density of the refrigerant at the suction state. The refrigerant mass flow rate is obtained from:

$$\dot{m}_r = V_r \rho_r - C\pi \tag{12}$$

where the last term represents the reduction of the mass flow rate due to the leakage. The refrigerant volumetric mass flow rate V_r in $[m^3/s]$ and the dimensionless coefficient C are inputs of the model.

2.2.2 Brine model

If the inputs of the model are obtained from a fluid in the evaporator and afterwards it is necessary to predict the heat pump behavior with a different fluid, for example if the inputs are estimated with water and predicted with brine, a model is necessary. Following [5], the global heat transfer coefficient can be obtained from:

$$UA_e = \frac{1}{\frac{C_3}{D_f}(\frac{\dot{m}_e}{\rho})^{-0.8} + C_2} \tag{13}$$

where the coefficients C_2 and C_3 are inputs of the model. The degradation factor D_f can be calculated as shown in [5]. When the fluid running through the evaporator is the same for the estimation and for the prediction mode, as in the present case, the D_f is equal to unity. However, the brine model is still used because the UA depends on $\dot{m}$, it is calculated from two parameters and the estimation procedure is more accurate when more parameters are employed. The same procedure can be used for the condenser, but in the present work this model is only applied for the evaporator side. Unfortunately, a validation of this model when $D_f \neq 1$ is not provided because no experimental data are available.
Summing up, the RC based model needs eight inputs C_2, C_3, UA_c, ΔT_{sh}, η, W_{loss}, ν_i and V_r that are to be obtained by multidimensional minimization algorithms. In the present paper these data are obtained from experiments using a Simplex Nelder minimization algorithm from GSL [4].

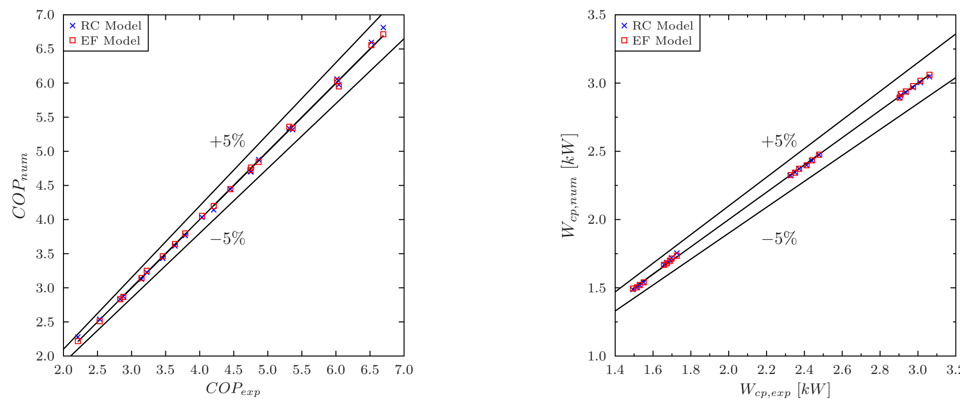
3 Results

In order to validate the models, experimental data obtained at ISFH are employed. The experimental set-up has been described in [7]. Experiments have been conducted in four cases depending on the mass flow rate defined here in $[kg/h]$: case-A) $\dot{m}_c = 500$ and $\dot{m}_e = 1900$; case-B) $\dot{m}_c = 700$ and $\dot{m}_e = 1900$; case-C) $\dot{m}_c = 900$ and $\dot{m}_e = 1900$ and case-D) $\dot{m}_c = 700$ and $\dot{m}_e = 1000$. Numerical calculations have been obtained with all possible combinations. For example, the parameters have been estimated at conditions of case-A and predicted in all conditions from case-A to case-C.
Experimental inlet fluid condenser temperatures range from 14^oC to 50^oC and inlet fluid evaporator temperatures from -5^oC to 30^oC with overlapping regions. The heat pump investigated has a scroll compressor with R410A as a refrigerant and the brine fluid of the evaporator side is $Tyfocor^{®}$.
In this work the experimental data are referred as non-standard conditions when the measured inlet temperature difference between the condenser and evaporator, $\Delta T_{diff} < 5^oC$ or when $T_{fe,i} > 20^oC$. All the other data are considered to be at standard conditions that represents the data typically provided by commercial catalogues.

3.1 Validation at standard conditions

For the validation procedure of this section, only the cases were the prediction mode is the same than that of the estimation mode are considered. Moreover, only experimental standard data are used. Numerical results compared agains experimental data calculated at case-A are shown in Fig.1a for the coefficient of performance (COP) and in Fig.1b for the compressor work W_{cp}. A relative error line, calculated as $\epsilon_r = 100 \cdot |(\phi_{num} - \phi_{exp})/\phi_{exp}|$ being ϕ is a generic variable, equal to 5% and to -5% are also plotted in Fig.1 for comparison purposes. In this case, both models predict experimental data with very satisfactory results with ϵ_r below 5%. In Table 1, the RMS (root mean square) error of all standard data are presented along with the maximum relative error $\epsilon_{r,max}$ for Q_c, W_{cp} and COP. In this section only the data of the Table 1 with the same mass flow rates in the estimation and prediction mode are considered. The RMS and the $\epsilon_{r,max}$ predicted for the EF model is always lower than that of the RC based model. Numerical results presented in Table 1 have been obtained for all mass flow rates used in the experiments, but only some data are presented in this work. The analysis of all data for the cases studied in this section, does not provide a significant difference from the analysis of data shown in Table 1. All studies lead to the observation that the RC model predictions are typically below 10% while EF errors are always below 5%.

This conclusion is supported by our previous study [3] where catalogue data from several heat pumps were used for the comparison. For steady state calculations where the boundary conditions are equal in the estimation and prediction mode, the EF is recommended. The EF model is more accurate and it can adjust to any brine to water heat pump easily.



(a) (b)

Figure 1: Numerical results of a) COP and b) W_{cp}, compared with experimental data at case-A. Model inputs obtained from same conditions than that of experiments.

One of the reasons of the better accuracy of the EF model is because it uses 12 parameters in the fitting procedure while the RC model is using only 8 inputs. To the author's opinion, a RC based model with 12 inputs will probably be as accurate as the EF model. However, the implementation of the RC model is much more complicated compared to the EF, specially for the model's input estimation procedure. Moreover, the algorithm used here to estimate the inputs of the RC model can not ensure the minimum absolute error, but only a relative. Therefore, the minimization process may change depending on initial values and some numerical parameters of the algorithm, which difficult the task of developing a robust tool to estimate the inputs. On the contrary, the input parameters of the EF model are much easier to be obtained and the estimation procedure does not depend on initial values and numerical parameters. Besides these, the RC model can only be accurate if the heat pump physical phenomena is considered. For example, a double circuit heat pump can be predicted with the present model but a higher errors than the ones shown here are obtained (see [3]).

It is also important to notice that if the RC based model is used in order to accurately match internal data of the heat pump, the inputs of the parameters can not be estimated from catalogue data, since a good prediction of Q_c and W_{cp} does not mean an accurate prediction of evaporative and condensing

Table 1: Root mean square (RMS) and maximum relative error $\epsilon_{r,max}$ of global variables as a function of the mass flow rates.

Estimation mode		**Prediction mode**			RMS			$\epsilon_{r,max}$		
$\dot{m}_c$	$\dot{m}_e$	$\dot{m}_c$	$\dot{m}_e$	Model	Q_c	W_{cp}	COP	Q_c	W_{cp}	COP
$[kg/h]$	$[kg/h]$	$[kg/h]$	$[kg/h]$		[%]	[%]	[%]	[%]	[%]	[%]
500	1900	500	1900	RC	9.75	1.21	4.60	2.82	1.64	3.06
(Case-A)		(Case-A)		EF	4.08	0.82	2.72	1.10	0.95	1.50
		700	1000	RC	79.89	12.30	91.85	19.23	12.36	36.04
		(Case-D)		EF	173.15	7.38	128.39	33.58	7.51	44.43
900	1900	500	1900	RC	49.73	2.49	22.25	6.79	2.49	7.02
(Case-C)		(Case-A)		EF	51.70	16.13	48.90	8.37	9.24	15.83
		900	1900	RC	23.97	1.74	15.13	7.28	2.04	8.00
		(Case-C)		EF	10.90	1.17	8.80	2.72	1.60	4.22

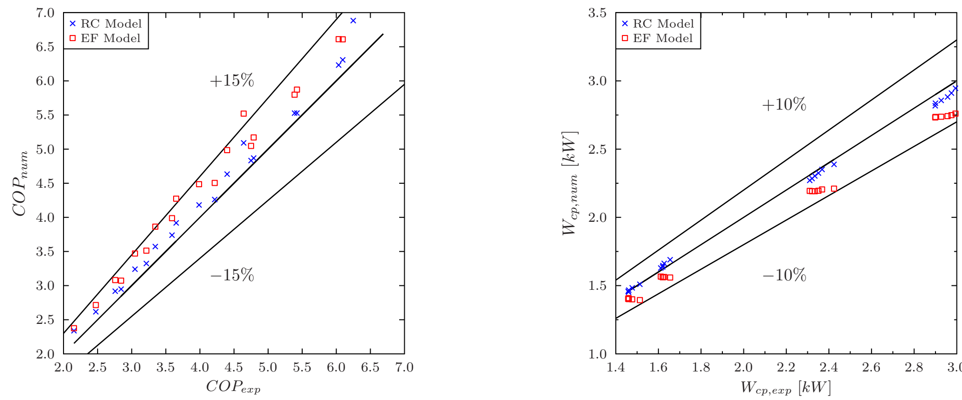
pressures, for example. The model was developed [2] to calculate global data such as Q_c, Q_e and W_{cp}, thereby internal heat pump data may not be accurately predicted using the present model without further improvements.

3.2 Mass flow rate analysis

Comparisons between the models for different mass flow rates in the evaporator and in the condenser have been analyzed. Predicted COP for inputs estimated at case-A and predicted at case-C have been plotted as a function of experimental data in Fig.2. In this case, predictions of both models are not as accurate as shown in the previous section with COP ϵ_r up to 15% for the EF model. The RC based model performs better than the EF model, which is something one might expect because the model is derived from physical concepts. All combination of cases from A to C have been studied but only some data are presented in Table 1. These results show that both RMS and $\epsilon_{r,max}$ are usually better predicted by the RC compared to the EF model. Analysing all combination of cases defined in section 3 with different mass flow rates in the estimation and prediction mode, a general conclusion can be drawn: the greater the difference between the mass flow rate used for estimation and prediction modes, the greater the error of the models and also the larger the difference between them (in favor of the RC model). Results presented in this section confirm the generalized opinion that RC based models tend to extrapolate better. Moreover, the implementation of Eq.13 for the condenser side should improve RC predictions for varying mass flow rate.

3.3 Model analysis at non-standard conditions

As explained in section 3, the experimental data obtained have been splitted into standard and non-standard data. The standard data have been used for estimation and prediction modes in all cases analyzed previously. In this section, the non-standard data have been used to analyse the behavior of the models under these conditions. Two cases have been considered: i) the standard data are used for estimation and the non-standard data for prediction and ii) all experimental data, including the non-standard values, are used for estimation procedure and the non-standard data are employed in the prediction mode. The first case is the most important, since typical catalogue data only include standard data and non-standard conditions may be found in system simulation calculations.



(a) (b)

Figure 2: Numerical results of a) COP and b) W_{cp} compared with experimental data at case-C. Model inputs obtained from experiments at case-A.

Numerical RMS and $\epsilon_{r,max}$ for Q_c and W_{cp} have been presented in Table 2 for the two cases studied here. Surprisingly, the EF model extrapolates better to non-standard conditions in the two cases analyzed. Nevertheless, when the estimate procedure is only using standard data none of the models provide satisfactory results and relative errors up to 30% can be found.

Table 2: Root mean square (RMS) and maximum relative error $\epsilon_{r,max}$ for predictions of non-standard data. Model inputs estimated at same conditions used in the prediction mode using only standard data or all experimental data for the estimating procedure.

			Using only standard data				**Using all experimental data**			
			RMS		$\epsilon_{r,max}$		RMS		$\epsilon_{r,max}$	
$\dot{m}_c$	$\dot{m}_e$	Model	Q_c	W_{cp}	Q_c	W_{cp}	Q_c	W_{cp}	Q_c	W_{cp}
$[kg/h]$	$[kg/h]$		[%]	[%]	[%]	[%]	[%]	[%]	[%]	[%]
500	1900	RC	228.56	8.21	33.82	8.90	129.40	5.99	22.19	6.95
(Case-A)		EF	185.64	5.88	27.93	5.18	21.48	1.08	2.69	0.94
900	1900	RC	177.84	16.50	29.69	11.52	130.77	3.00	24.21	3.30
(Case-C)		EF	95.81	2.68	15.59	1.85	9.49	1.85	1.30	1.92

The EF model performs very well if the non-standard data are used in the estimation procedure, with errors in the same range of accuracy as results presented in section 3.1. However, the RC predictions are not satisfactory even when all data for the estimation procedure are employed. For example, $\epsilon_{r,max}$ of 34% in Q_c calculations are observed. When the compressor pressure ratio decreases because the evaporator and condenser inlet temperatures are close to each other, the COP increases until a certain point where the performance stabilizes (see [7]). This phenomena can be considered in the EF model if non-standard data are used for the fitting procedure. However, it is not considered in the mathematical description of the compressor of the RC based model. Therefore, if non-standard conditions have to be well predicted, the compressor model of the RC based approach should consider the compressor performance decrease at low pressure ratios.

4 Conclusions

An equation fit (EF) and a refrigerant circuit (RC) based heat pump models have been described, validated through comparisons agains experimental data, analyzed and compared to each other for varying mass flow rate and under non-standard conditions. From this work, the following conclusions can be drawn:

- When the same boundary conditions are used in the estimation and prediction mode, clearly, the EF model performs better and it is recommended, not only for its better accuracy, but also because the inputs of the model are much more easier to fit and the model is easier to implement.
- The RC based model extrapolates better when the mass flow rate is different in the prediction mode with respect the one employed in the estimation mode.
- The EF model extrapolates better for non-standard conditions. If the fitting procedure is done using non-standard data, the EF model would be as accurate as in standard conditions. Otherwise, $\epsilon_{r,max}$ of Q_c in the range of 16% can be expected. For the RC model, even using non-standard data for estimating the inputs, high errors, with $\epsilon_{r,max}$ up to 35% for Q_c, may be found.

Acknowledgments

This work has been partially supported by the Spanish Ministerio de Ciencia e Innovación (project reference ENE 2010-18994).

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